Linear stability characterization of thin viscoelastic liquid films flowing down a plate moving in a vertical direction

Hung Ming Sung, Zong-Yi Lee, Chung-Ting Hsu

Abstract

This project presents a stability analysis of thin viscoelastic liquid films flowing down a plate moving in a vertical direction. The long-wave perturbation method is employed to derive the generalized kinematic equations for a free film interface. The current thin liquid film stability analysis provides a valuable input to investigations into the influence of the style of motion of the vertical plate on the stability behavior of the thin film flow.

Keywords: viscoelastic liquid, thin film flow, long-wave perturbation.

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沿垂直方向移動之直立平板表面流下的黏彈性流體薄膜流的線性穩定性分析

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摘 要

本文針對黏彈性流體薄膜流，探討沿垂直方向移動的直立平板表面流下的薄膜流之線性液動穩定性問題。首先使用長波微擾法推導薄膜的自由面方程式，在探討薄膜流場的穩定性問題上，主要分析平板的移動效應對系統穩定性的影響。

關鍵詞：黏彈性流體、薄膜流、長波微擾法。

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1. Introduction

The stability characterization of film flows traveling down along a vertical or an inclined plate is of great importance to the quality control of many industrial products. Thus, the research effort made toward improvement on this matter has been emerged as a subject of great interest to numerous worldwide researchers in past decades. Typical application examples can be found across different industrial sectors including mechanical, chemical and nuclear engineering. It is well known that the stability controls are generally required in precision finishing processes of coating, laser cutting, and casting. Since macroscopic instability can cause disastrous conditions to film flows and thus very detrimental to the needed quality of final products, it is highly desirable to develop suitable working conditions for homogeneous film growth to adapt to various flow configurations and associated time-dependent properties.

Detailed reviews on linear stability theories for various film flows has formally presented by Lin [1] and Chandrasekhar [2]. The Landau equation was re-derived in 1956 by Stuart [3] using the disturbed energy balance equation and Reynolds stresses. Benjamin [4] and Yih [5] formulated the perturbed wave equation for free surface flows. The stability behaviors of flows having long disturbed wave were arefully studied in this paper and some significant observations on film flows over an inclined plane are obtained. These observations include (1) the flow that is disturbed by a longer wave is less stable than that of the flow disturbed by a shorter wave; (2) the film flow becomes less stable as the inclined angle increases; (3) the film flow traveling down along a vertical plate becomes unstable as the critical Reynolds number becomes nearly zero; (4) the film flow becomes somehow stabilized as the surface tension of the film increases; (5) velocity of the unstable long disturbed wave is approximately twice of the wave velocity on the free surface. The effect of surface tension was found by many researchers [6-8] as one of the necessary conditions that lead to the solution of supercritical stability in analyzing this type of problems. The effect of surface tension on flow stability was considered significant by Lin [6], Nakaya [7], and Krishna et al. [8]. Renardy et al. [9] and Tsai et al. [10] presented the work of both linear and nonlinear stability analysis for a film flow traveling down along an inclined or a vertical plate. Detailed flow analysis was found of great importance in the development of stability theory for characterizing the behaviors of various film flows. Andersson et al. [11] studied the gravity-driven flow of a viscoelastic film flow traveling down along a vertical wall. The derived analytical expression of film thickness reveals that the film thickness of a viscoelastic film can develop more rapidly than that of the Newtonian film in downstream asymptotic states. Walters [12] analyzed the motion behavior of a viscoelastic film flow that is confined in between two coaxial cylinders. Cheng et al. [13] studied the stability of thin viscoelastic film flow traveling down along a vertical wall. The results of their studies indicate that the viscoelastic parameter indeed plays a significant role in destabilizing the film flow.

After careful literature review on the papers of thin viscoelastic film flows raveling down along a vertical plate, it was found that the stability of thin viscoelastic film flows moving along vertical plates appeared to be very important in various coating, painting, surface drawing and lubrication processes. This type of stability problems has not yet been fully explored so far in the literature. The types of stability problems are indeed of great importance for many industrial applications. In this paper, the finite-amplitude stability of a thin viscoelastic film flow traveling down along a vertical quiescent, up-moving, and down-moving plate is thoroughly investigated. The influence of the plate moving styles on the equilibrium finite amplitude is studied and characterized. Several numerical examples are presented to verify the computational results and also to illustrate the effectiveness of the proposed modeling approach.

2.Generalized Kinematic Equation

Fig.1 shows the configuration of a thin viscoelastic film flow traveling down along a vertically moving plate. The fluid used for study is an incompressible viscoelastic prototype that is designated as liquid B'' by Beard and Walters[14]. The Walters' liquid B'' represents an approximation to the first order in elasticity, i.e. for short or rapidly fading memory fluids. All associated physical properties and the rate of film flow are assumed to be constant (i.e. time-invariant). Based on the given assumptions, the velocity fields of the film flow can be represented by

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \quad (1)$$

$$\rho(\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}) = \rho g+\frac{\partial \tau_{x^*x^*}}{\partial x^*}+\frac{\partial \tau_{y^*x^*}}{\partial y^*} \quad (2)$$

$$\rho(\frac{\partial v^*}{\partial t^*}+u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}) = \frac{\partial \tau_{x^*y^*}}{\partial x^*}+\frac{\partial \tau_{y^*y^*}}{\partial y^*} \quad (3)$$

where ρ is the density of the film flow. Individual stress components can be expressed in terms of velocity gradient and flow pressure as

$$\tau_{x^*x^*}=-p^*+2\mu\frac{\partial u^*}{\partial x^*}-2k_0[\frac{\partial^2 u^*}{\partial t^*\partial x^*} +u^*\frac{\partial^2 u^*}{\partial x^{*2}}+v^*\frac{\partial^2 u^*}{\partial x^*\partial y^*}-2(\frac{\partial u^*}{\partial x^*})^2 -\frac{\partial u^*}{\partial y^*}(\frac{\partial u^*}{\partial y^*}+\frac{\partial v^*}{\partial x^*})] \quad (4)$$

$$\tau_{y^*y^*}=-p^*+2\mu\frac{\partial v^*}{\partial y^*}-2k_0[\frac{\partial^2 v^*}{\partial t^*\partial y^*} +v^*\frac{\partial^2 v^*}{\partial y^{*2}}+u^*\frac{\partial^2 v^*}{\partial x^*\partial y^*}-2(\frac{\partial v^*}{\partial y^*})^2 -\frac{\partial v^*}{\partial x^*}(\frac{\partial v^*}{\partial x^*}+\frac{\partial u^*}{\partial y^*})] \quad (5)$$

$$\tau_{x^*y^*}=\tau_{y^*x^*}=\mu(\frac{\partial u^*}{\partial y^*}+\frac{\partial v^*}{\partial x^*})-k_0[\frac{\partial^2 u^*}{\partial t^*\partial y^*} +\frac{\partial^2 v^*}{\partial t^*\partial x^*}+u^*(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial x^*\partial y^*}) +v^*(\frac{\partial^2 v^*}{\partial x^*\partial y^*}+\frac{\partial^2 u^*}{\partial y^{*2}}) -2\frac{\partial u^*}{\partial y^*}\frac{\partial v^*}{\partial y^*}-2\frac{\partial v^*}{\partial x^*}\frac{\partial u^*}{\partial x^*}] \quad (6)$$

where u^* and v^* are velocity components in x^* and y^* directions, respectively. p is the flow pressure, ρ is the film density, and μ is the dynamic viscosity. The boundary conditions for the film flow system at the plate surface of $y^*=0$ can be expressed as

$$u^*=U^* \quad (7)$$

$$v^*=0 \quad (8)$$

where U^* is the moving velocity of the vertical plate. The boundary conditions for the film flow at free surface of $y^*=h^*$ are derived based on the results given by Edwards et al. [15]. The shear stress for film flow at free surface is given as

$$\frac{\partial h^*}{\partial x^*}[1+(\frac{\partial h^*}{\partial x^*})^2]^{-1}(\tau_{y^*y^*}-\tau_{x^*x^*}) +[1-(\frac{\partial h^*}{\partial x^*})^2][1+(\frac{\partial h^*}{\partial x^*})^2]^{-1}\tau_{x^*y^*}=0 \quad (9)$$

The normal stress for film flow at free surface is given as

$$[1+(\frac{\partial h^*}{\partial x^*})^2]^{-1}[2\tau_{x^*y^*}\frac{\partial h^*}{\partial x^*}-\tau_{y^*y^*} -\tau_{x^*x^*}(\frac{\partial h^*}{\partial x^*})^2]+S^*\{\frac{\partial^2 h^*}{\partial x^{*2}}[1 +(\frac{\partial h^*}{\partial x^*})^2]^{-3/2}\}=Pa^* \quad (10)$$

The kinematic condition that the flow velocity normal to a free surface is naught can be given as

$$\frac{\partial h^*}{\partial t^*}+\frac{\partial h^*}{\partial x^*}u^*-v^*=0 \quad (11)$$

where p_a^* is the ambient pressure, S^* is the surface tension, h^* is the local film thickness. The variable associated with a superscript " * " stands for a dimensional quantity. By introducing the stream function φ^*, the dimensional velocity components can now be expressed as

$$u^* = \frac{\partial \varphi^*}{\partial y^*}, \quad v^* = -\frac{\partial \varphi^*}{\partial x^*} \tag{12}$$

In order to minimize the flow variables and to simplify the analysis procedure, it is customary to define dimensionless variables as

$$x = \frac{\alpha x^*}{h_0^*}, \quad y = \frac{y^*}{h_0^*}, \quad t = \frac{\alpha u_0^* t^*}{h_0^*},$$

$$h = \frac{h^*}{h_0^*}, \varphi = \frac{\varphi^*}{u_0^* h_0^*}, p = \frac{p^* - p_a^*}{\rho u_0^{*2}}$$

$$\mathrm{Re} = \frac{u_0^* h_0^*}{\nu}, S = \left(\frac{S^{*3}}{2^2 \rho^3 \nu^4 g}\right)^{1/3}$$

$$\alpha = \frac{2\pi h_0^*}{\lambda}, u_0^* = \frac{g h_0^{*2}}{2\nu},$$

$$k = \frac{k_0}{\rho h_0^{*2}} \tag{13}$$

The moving velocity of the vertical plate can then be expressed as

$$U^* = Z u_0^* \tag{14}$$

where Z is a specific constant ratio of the plate velocity to the free stream velocity.

Since the modes of long-wavelength that gives the smallest wave number are most likely to induce flow instability for the film flow [4,5], the dimensionless wave number of the long-wavelength mode, α, is then chosen as the perturbation parameter for variable expansion. By so doing the stream function and flow pressure can be perturbed and represented as

$$\varphi = \varphi_0 + \alpha \varphi_1 + O(\alpha^2) \tag{15}$$

$$p = p_0 + \alpha p_1 + O(\alpha^2) \tag{16}$$

In practice, the non-dimensional surface tension S is a large value. The term $\alpha^2 S$ can be treated as a quantity of zero-th order [10]. The generalized nonlinear kinematic equation can be obtained as

$$\begin{aligned} & h_t + A(h)h_x + B(h)h_{xx} + C(h)h_{xxxx} \\ & + D(h)h_x^2 + E(h)h_x h_{xxx} = 0 \end{aligned} \tag{17}$$

where

$$A(h) = \frac{2}{1+Z} h^2 + \frac{Z}{1+Z} \tag{18}$$

$$\begin{aligned} B(h) = \alpha\, \mathrm{Re}[& \frac{8}{15} \frac{1}{(1+Z)^2} h^6 \\ & + \frac{4}{3} k \frac{1}{(1+Z)^2} h^4] \end{aligned} \tag{19}$$

$$C(h) = \frac{2}{3} \mathrm{Re}^{-\frac{2}{3}} S \alpha^3 (1+Z)^{-1/3} h^3 \tag{20}$$

$$\begin{aligned} D(h) = \alpha\, \mathrm{Re}[& \frac{16}{5} \frac{1}{(1+Z)^2} h^5 \\ & + \frac{16}{3} k \frac{1}{(1+Z)^2} h^3] \end{aligned} \tag{21}$$

$$E(h) = 2\, \mathrm{Re}^{-\frac{2}{3}} S \alpha^3 (1+Z)^{-1/3} h^2 \tag{22}$$

In order to characterize more precisely the effect of vertical plate motion on the stability behaviors of a down-traveling thin film flow, a detailed numerical investigation on flow stability is carried out. Three different kinds of plate-moving styles, i.e. stationary, up-moving, and down-moving movements, for various speeds are used to characterize the behaviors of stable thin film flows traveling down along the moving plate. The flow rate of the film flow is assumed to be constant. The variations of local film thickness and the flow velocity at free surface in equilibrium are defined as

$$u_0^* = (1+Z)\left(\frac{\frac{5}{24}}{\frac{5}{24}+\frac{Z}{4}}\right)^{1/2}\bar{u}_0^* \qquad (23)$$

$$h_0^* = \left(\frac{\frac{5}{24}}{\frac{5}{24}+\frac{Z}{4}}\right)^{1/4}\bar{h}_0^* \qquad (24)$$

where $\bar{u}_0^*$ is the velocity at free surface for a static plate in equilibrium state, and $\bar{h}_0^*$ is the film thickness in equilibrium state when the plate is static.

3. Stability Analysis

The dimensionless film thickness when expressed in perturbed state can be given as

$$h(x,t) = 1 + \eta(x,t) \qquad (25)$$

where η is a perturbed quantity of stationary film thickness. By inserting equation (25) into equation (17) and collecting all terms up to the order of η^3, the evolution equation of η becomes

$$\begin{aligned}&\eta_t + A\eta_x + B\eta_{xx} + C\eta_{xxxx} + D\eta_x^2 + E\eta_x\eta_{xxx} \\ &= -[(A'\eta + \frac{A''}{2}\eta^2)\eta_x + (B'\eta + \frac{B''}{2}\eta^2)\eta_{xx} \\ &\quad + (C'\eta + \frac{C''}{2}\eta^2)\eta_{xxxx} + (D + D'\eta)\eta_x^2 \\ &\quad + (E + E'\eta)\eta_x\eta_{xxx}] + O(\eta^4)\end{aligned} \qquad (26)$$

where all the values of A, B, C, D, E and their derivatives are evaluated at the dimensionless film height of the film h=1.

To characterize the linear behaviors of the film flow, the nonlinear terms in equation (26) are assumed insignificant and can be neglected to obtain the linearized equation

$$\eta_t + A\eta_x + B\eta_{xx} + C\eta_{xxxx} = 0 \qquad (27)$$

The normal mode analysis [16] can be performed by assuming that

$$\eta = a\exp[i(x - dt)] + c.c. \qquad (28)$$

where a is the perturbed wave amplitude, and c.c. is the associated complex conjugate counterpart. The complex wave celerity, d, can be expressed as

$$d = d_r + id_i = A + i(B - C) \qquad (29)$$

where d_r is the linear wave speed, and d_i is the linear growth rate of the wave amplitudes. The flow is linearly unstable supercritical if $d_i > 0$, and is linearly stable sub-critical if $d_i < 0$.

4. Numerical Illustrations and Discussions

A numerical example is presented here to illustrate the effectiveness of the proposed modeling approach for characterizing the thin viscoelastic film flow traveling down along a vertically moving plate. In order to reliably verify the results of theoretic derivation, a finite amplitude perturbation apparatus is used to numerically generate the needed perturbation parameters for linear stability analyses. It is obvious from the nonlinear kinematic equation that the stability of a thin-film flow is closely related and can be characterized by several flow variables including Reynolds number, Re, velocity ratio of the plate to free stream, Z, viscoelastic parameter, k, and dimensionless perturbation wave number, α . Some important features appeared in modeling results are carefully extracted and used to compare with some conclusive results given in the literature.

Fig. 1 shows the schematic diagram of a thin viscoelastic film flow traveling down along a vertically down-moving upright plate. Physical parameters that are selected for study include (1) Reynolds numbers ranging from 0 to 15, (2) the dimensionless perturbation wave numbers ranging from 0 to 0.12, (3) the value of viscoelastic parameter is given as 0.02[13], and (4) the velocity ratios Z for use in this study include –0.42, -0.32, -0.18, 0, 0.23, 0.51, 0.85. A constant dimensionless surface tension value is given for computation to enable the study of film flow stability behaviors for different plate-moving conditions of moving-up (Z = -0.42, -0.32, -0.18), stationary (Z = 0), and moving-down (Z = 0.23, 0.51, 0.85). In other words, S is selected as 6173.5 [13]. This value is selected here for study mainly for comparing the final result with data given in the literature. It is found that the results obtained by using the proposed method for the thin viscoelastic film flow traveling down along a stationary vertical plate (i.e. Z = 0) agree well with those data given by Cheng et al. [13].

The linear neutral stability curve is obtained by setting $d_i = 0$ in equation (29). The α -Re plane is divided into two different characteristic regions by the neutral stability curve. One is the linearly stable region where small disturbances decay with time and the other is the linearly unstable region where small perturbations grow as time increases. The linear neutral stability curves of a thin viscoelastic film flow traveling down along a vertical plate for three different plate-moving conditions are computed. The results are presented in Fig. 2(a) and 2(b). Fig. 2(a) shows the linear stability curves of the film flow traveling down along a down-moving plate for Z=0, 0.23, 0.51, 0.85. The results indicate that the linear stable region $(d_i < 0)$ is enlarged as the velocity of the down-moving plate increases. On the other hand, the linear stability curves of the film flow traveling down along a up-moving plate for Z=0, -0.18, -0.32, -0.42 are presented in Fig. 2(b). The results indicate that the linear stable region shrinks as the velocity of the up-moving plate increases. It becomes quite obvious from the numerical experiments given in Fig. 2(a) that the down-moving motion of the vertical plate tends to enhance the stability of the down-traveling film flow on the plate. It is also true by observations from Fig. 2(b) that the up-moving motion of the vertical plate tends to destabilize the down-traveling film flow on the plate.

The temporal amplitude growth rate of the disturbed wave is also computed by using equation (29). The results are presented in Fig. 3 and 4. Fig. 3(a) shows the temporal amplitude growth rates of the disturbed wave for various perturbed wave numbers, α's and at different down-moving plate velocity ratios of Z=0, 0.23, 0.51, 0.85 at Reynolds number Re=10. The temporal amplitude growth rates of the disturbed wave for various Reynolds numbers, Re, with different down-moving plate velocity ratios of Z=0, 0.23, 0.51, 0.85 for a perturbed wave number of α=0.06 are given in Fig. 3(b). It is found that the amplitude growth rate of the disturbed wave, d_i, decreases as the down-moving plate velocity increases. The decreasing rate of d_i tends to slow down as the down-moving plate velocity becomes larger. The results indicate that the down-moving motion of the vertical plate tends to enhance the stability of the down-traveling film flow on the plate. It is also obvious from Fig. 3(a) and 3(b) that the increased enhancement on flow stability is more distinct for a smaller down-moving plate velocity than that of the larger down-moving plate velocity. Fig. 4(a) shows the temporal amplitude growth rates of the disturbed wave for various perturbed wave numbers, α's and different up-moving plate velocity ratios of Z=0, -0.18, -0.32, -0.42 at Reynolds number Re=10.

The temporal amplitude growth rates of the disturbed wave for various Reynolds numbers, Re, with different up-moving plate velocity ratios of Z=0, -0.18, -0.32, -0.42 for a perturbed wave number of α=0.06 are given in Fig. 4(b). It is found that the amplitude growth rate of the disturbed wave, d_i, increases as the up-moving plate velocity increases. The increasing rate of d_i tends to increase more substantially as the up-moving plate velocity increases. The results indicate that the up-moving motion of the vertical plate tends to destabilize the down-traveling film flow on the plate. It is also obvious from Fig. 4(a) and 4(b) that the destabilization effect of the down-traveling flow on a up-moving plate is more distinct for a larger up-moving plate velocity than that of the smaller up-moving plate velocity.

5. Conclusion

The stability of a thin viscoelastic film flow traveling down along a vertical plate under three different plate moving conditions is investigated by using the method of long-wave perturbation. The generalized nonlinear kinematic equations of the film flow at the interface of free surface is derived and numerically estimated to characterize the behaviors of flow stability. Based on the results of numerical modeling, several conclusions can be drawn as follows:

(1) The neutral stability curve obtained by using linear stability analysis separates the α-Re plane into two different characteristic regions. It is interesting to note that as the down-moving plate velocity increases, the linear stable region also increases, however, the temporal film growth rate of the perturbed wave decreases. In other words, the film flow becomes more stable as the down-moving plate velocity increases.

(2) When the up-moving plate velocity increases, the linear stable region decreases, however, the temporal film growth rate of the perturbed wave increases. In other words, the film flow gradually becomes unstable when the up-moving plate velocity increases.

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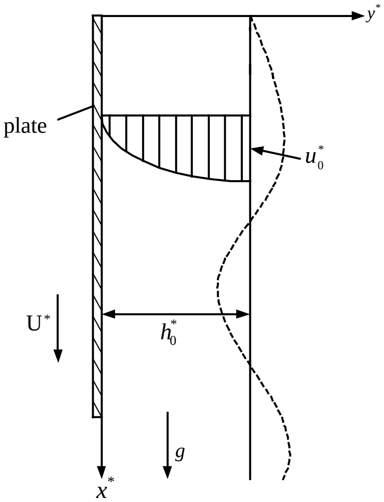

Fig. 1 Schematic diagram of a thin viscoelastic film flow traveling down along a vertically moving upright plate

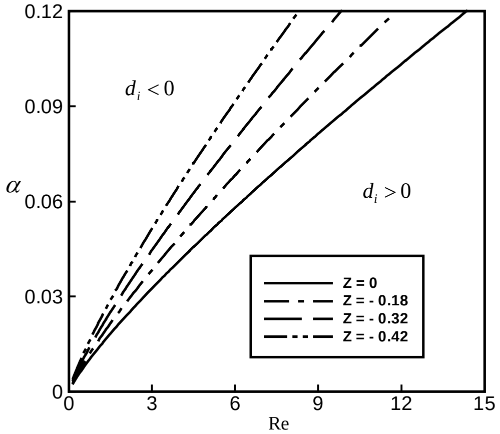

Fig. 2(b) Neutral linear stability curves of a viscoelastic flow for different up-moving plate velocity ratios, Z's

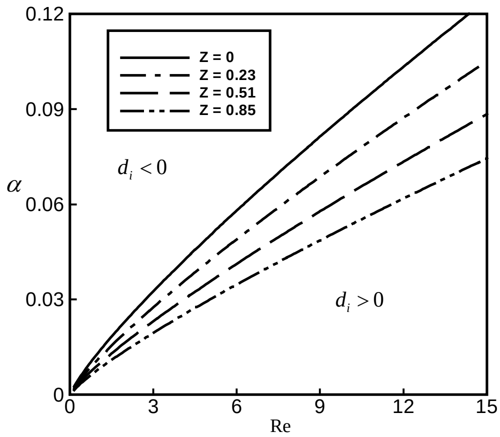

Fig. 2(a) Neutral linear stability curves of a viscoelastic flow for different down-moving plate velocity ratios, Z's

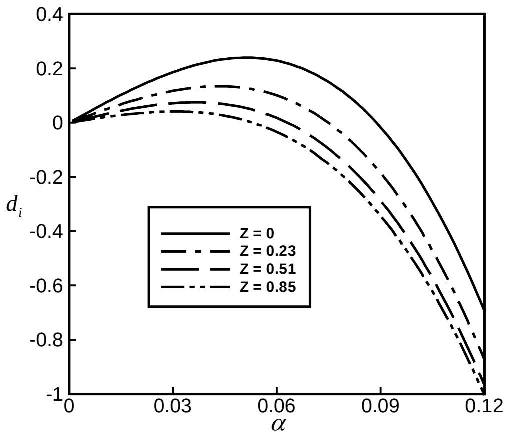

Fig. 3(a) Linear amplitude growth rate of disturbed waves in a viscoelastic flow for different down-moving plate velocity ratios, Z' s, at Re=10

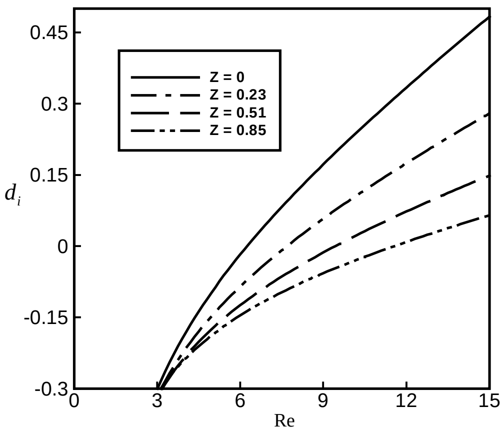

Fig. 3(b) Linear amplitude growth rate of disturbed waves in a viscoelastic flow for different down-moving plate velocity ratios, Z' s, at $\alpha = 0.06$

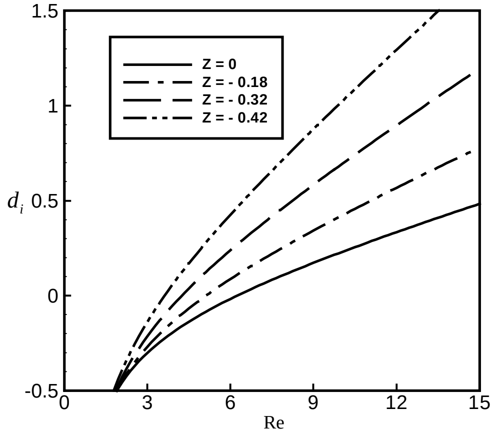

Fig. 4(b) Linear amplitude growth rate of disturbed waves in a viscoelastic flow for various up-moving plate velocity ratios, Z' s, at $\alpha = 0.06$

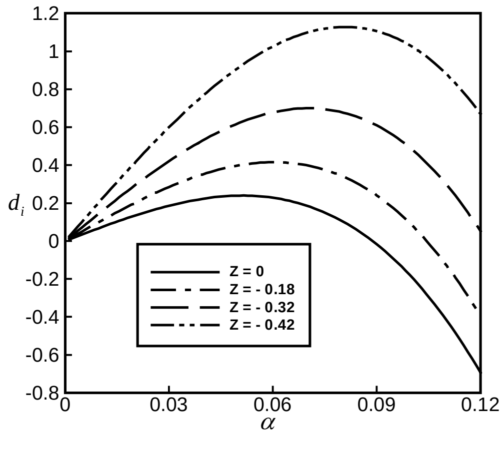

Fig. 4(a) Linear amplitude growth rate of disturbed waves in a viscoelastic flow for different up-moving plate velocity ratios, Z' s, at Re=10